100 Probability Questions

Prob-Stats (Math 3350)

Formulas and Notation

- **Cardinality Operator**: \( n(S) \), counts the (possibly infinite) number elements in set \( S \).
- **Probability**: \( P(A) = \frac{n(A)}{n(S)} \), the probability of event \( A \) is the cardinality of event set \( A \) divided by the cardinality of the sample space \( S \).
- **Permutations**: \( P(n, k) = \frac{n!}{(n-k)!} \), the number of ordered ways to permute \( n \) objects into \( k \) bins.
- **Combinations**: \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \), the number of unordered ways to arrange \( n \) objects into \( k \) bins. This notation also refers to the binomial coefficients and is read “\( n \) choose \( k \)”.
- **We will use “word” to refer to all possible distinct orderings of a group of letters without concern for whether or not these orderings would show up in a dictionary.**

Problems

1. A standard dice is rolled. What is the probability that a 2, 4, OR 6 will be rolled?
   \[ P = \frac{1}{2} \]
2. Rosa will toss a fair coin twice. If you know that the first coin toss resulted in heads, what would the probability be that both coins would land on heads?
   \[ P = \frac{1}{2} \]
3. A spinner is divided into 3 equal sections, with sections labeled 1, 2, and 3. What is the probability of spinning a 3 on the spinner if you know the arrow landed on an odd number?
   \[ P = \frac{1}{2} \]
4. A party host gives a door prize to one guest chosen at random. There are 48 men and 42 women at the party. What is the probability that the prize goes to a woman?
   \[ P = \frac{42}{90} = \frac{7}{15} \]
5. A spinner is divided into five equal sections numbered 1 through 5. The arrow is equally likely to land on any section.
(a) Find the probability table, which is also called the discrete “probability density function” (or pdf).

(b) Calculate the expected value $E(X)$ (or mean $\mu_x$) of the probability experiment.

(c) Calculate the variance $V(X)$ of the probability experiment.

(d) Calculate the standard deviation.

\[
E(X) = 1(\cdot2) + 2(\cdot2) + 3(\cdot2) + 4(\cdot2) + 5(\cdot2) = \frac{15}{5} = 3
\]

\[
E(X^2) = 1(\cdot2) + 4(\cdot2) + 9(\cdot2) + 16(\cdot2) + 25(\cdot2) = \frac{55}{5} = 11
\]

\[
V(X) = E(X^2) - E(X)^2 = 11 - 9 = 2
\]

\[
\sigma_x = \sqrt{V(X)} = \sqrt{2} \approx 1.41214
\]

We can use our graphing calculators to check our work as follows:

6. Alaska license plates have two letters followed by three numbers. What is the probability that a randomly chosen license plate will have an NC with the number ending in a 3?

\[
P(\text{NC} \quad 3) = \frac{100}{(26^2)(10^3)} = \frac{1}{6760}
\]

7. A 20-sided is rolled. If the result is 10 or less, the 20-sided die is rolled again. If the result is 11 or more, a 4-sided die is rolled. In either case, the results are summed after the second die roll. What is the probability that the sum will be 14?

If we list all the possible ways to get a 14, we can calculate the probabilities and sum them. Recall this only works because of the events listed below is independent.

- 1 then 13, with $P = (.05)^2$
- 2 then 12, with $P = (.05)^2$
- 3 then 11, with $P = (.05)^2$
- 11 then 3, with $P = (.05)(.25)$
• 12 then 2, with $P = (.05)(.25)$
• 13 then 1, with $P = (.05)(.25)$

$$P = 3(.05)^2 + 3(.05)(.25) = \frac{9}{200}$$

8. Police plan to enforce speed limits during the morning rush hour on four different routes into the city. The traps on routes A, B, C, and D are operated 40%, 30%, 20%, and 30% of the time, respectively. Biff always speeds to work, and he has probability 0.2, 0.1, 0.5, and 0.2 of using those routes.

(a) What is the probability that he’ll get a ticket on any one morning?

We must sum the probabilities of getting a ticket by the frequencies with which he travels each route: $P = .4(.2) + .3(.1) + .2(.5) + .3(.2) = .08 + .03 + .10 + .06 = .27$.

(b) What is the probability he’ll go five mornings without a ticket?

If he has a .27 chance of a ticket any given morning, he has a .73 chance of escaping without a ticket each morning. So the probability of him making it 5 straight days with no ticket is $.73^5 \approx .20731$

9. In an urn are 5 blue, 3 red, and 2 yellow marbles. If you draw 3 marbles, what is the probability that less than 2 will be red if:

(a) You draw with replacement?

The probabilities are fixed. So the probability of no red at all is $P(R = 0) = (.7)^3$

There are three ways to get one red - in the first draw, second draw or third draw. In all three cases, the probability is the same: $P(Rxx) = P(xRx) = P(xxR) = (.3)(.7)(.7)$.

So we add these totals to get the probability required.

(b) You draw without replacement?

So, 3 of the 10 marbles are red. The probability of drawing less than two is the sum of the probabilities of drawing either 1 or none:

$$P(R < 2) = \frac{{3\choose 1}{7\choose 2} + {3\choose 2}{7\choose 1}}{{10\choose 3}}$$

10. Butch will miss an important TV program while taking his statistics exam, so he sets both his VCRs to record it. The first one records 70% of the time, and the second one records 60% of the time. What is the probability that he gets home after the exam and finds?

We assume that events $A$ and $B$ are independent, so with $P(A) = .7$ and $P(B) = .6$ and their set complements $A^c$ and $B^c$ occurring with probabilities .3 and .4 respectively, we have:

(a) No copies of his program?

$$P(A^c \text{ and } B^c) = P(A^c)P(B^c) = (.3)(.4) = .12$$
(b) One copy of his program?

We have to account for both machines being the one that records, so

\[ P(A \text{ and } B^c) + P(A^c \text{ and } B) = P(A)P(B^c) + P(A^c)P(B) \]
\[ = (.7)(.4) + (.3)(.6) \]
\[ = .46 \]

Also note that part (b) is the only other possibility not covered in parts (a) and (c), so we could have taken those more easily calculated values, summed them, and subtracted from 1.

(c) Two copies of his program?

\[ P(A \text{ and } B) = P(A)P(B) = (.7)(.6) = .42 \]

(d) Good grief, a VCR? What’s that?

11. Two squares are chosen at random on a chessboard. What is the probability that they have a side in common?

Let’s attempt the following. Given any chess square, choose a different square at random. What is the probability it shares a side with the first square? (I’m not sure this is exactly what the question is asking, but it’s a probability I know I can calculate. This illustrates a good problem-solving technique known as solving a “simpler yet related” problem.) The 3 types of chess squares will require us to weight the probabilities by how often they occur. Consider that for our first square chosen, we have:

- Corner squares (4 total), each of which shares a side with only 2 other squares
- Edge squares (24 total, 6 per side), each of which shares sides with 3 other squares
- Middle squares (36 interior squares in a $6 \times 6$ grid), each of which shares sides with 4 other squares

<table>
<thead>
<tr>
<th>Type</th>
<th>Number</th>
<th>Frequency</th>
<th>Adjacent Squares</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner</td>
<td>4</td>
<td>$\frac{4}{64} = \frac{1}{16}$</td>
<td>2</td>
<td>$\frac{2}{63}$</td>
</tr>
<tr>
<td>Edge</td>
<td>24</td>
<td>$\frac{24}{64} = \frac{3}{8}$</td>
<td>3</td>
<td>$\frac{3}{63}$</td>
</tr>
<tr>
<td>Middle</td>
<td>36</td>
<td>$\frac{36}{64} = \frac{9}{16}$</td>
<td>2</td>
<td>$\frac{4}{63}$</td>
</tr>
</tbody>
</table>

The probabilities are multiplied the frequencies to weight them according to often they occur, and we get:
\[ P = \frac{42 + 243 + 364}{6463 + 6463 + 6463} = \frac{4(2) + 24(3) + 36(4)}{63(64)} = \frac{224}{4032} = \frac{1}{18} \]

I find it comforting that an answer accompanied the problem, and mine matches it precisely. This makes me feel more certain I understood the question, and my restatement of the problem above is what its original author intended.

12. At a baby shower, we started discussing baby statistics. One of the women told us she had heard a report that for every 100 babies born, there were 6 more boys than girls.

(a) If we were to randomly pick a child from a representative group, what is the probability of picking a girl?
We would have the following set: (47 girls, 53 boys) = 0.47

(b) If we were to pick 10 babies at random, what is the probability that at least half of them would be girls?
We can use the binomial theorem:
\[ P(G \geq 5) = \sum_{k=5}^{10} \binom{10}{k}(.47)^k(.53)^{10-k} \]

(c) If we were to pick 10 babies at random, what is the probability that exactly 8 of them would be girls?
\[ P(G = 5) = \binom{10}{8}(.47)^8(.53)^2 \]

(d) If we were to pick 10 babies at random, what is the probability that no more than 8 of them would be boys?
\[ P(B \leq 8) = 1 - P(B \geq 9) = 1 - \left[ \binom{10}{9}(.53)^9(.53) + (.53)^{10} \right] \]
13. There are an equal number of pennies, nickels, dimes, and quarters in a bag. What is the probability that the combined value of the four coins randomly selected with replacement will be $0.41?

This is equivalent to a 4-letter alphabet of P, N, D and Q. The question is, if we choose 4 letters at random with replacement, what is the probability all 4 letters are different? Thus:

\[ P = \frac{4!}{4^4} = \frac{4 \times 3 \times 2 \times 1}{4 \times 4 \times 4 \times 4} = \frac{3}{32} \]

As a side note, my answer doesn’t match the one I was given by the team that found it, yet I remain confident I’m correct. The denominator is straightforward. For the numerator, we know there is exactly one way to pick each of the 4 letters (or coin types). But order does not matter, so multiply by the 4! orderings that are possible.

14. A quarter, two dimes, a nickel and four pennies are placed in a bag and mixed thoroughly. Two coins are drawn at random without replacement, and their total monetary value is recorded. Create a probability table (pdf) for this probability experiment, and then calculate the expected value, variance and standard deviation.

<table>
<thead>
<tr>
<th>Coins</th>
<th>PP</th>
<th>PN</th>
<th>NN</th>
<th>DP</th>
<th>DN</th>
<th>DD</th>
<th>QP</th>
<th>QN</th>
<th>QD</th>
<th>QQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x) )</td>
<td>( \binom{1}{2} = \frac{6}{28} )</td>
<td>( \frac{4 \times 1}{28} )</td>
<td>0</td>
<td>( \frac{4 \times 2}{28} )</td>
<td>( \frac{2 \times 1}{28} )</td>
<td>( \frac{1}{28} )</td>
<td>( \frac{4 \times 1}{28} )</td>
<td>( \frac{1}{28} )</td>
<td>( \frac{2 \times 1}{28} )</td>
<td>0</td>
</tr>
</tbody>
</table>

Thankfully, the probabilities sum to 1, and we feel reasonably certain we may have calculated them accurately. Simplifying the table by getting rid of things that have no chance of happening, and doing a bit of arithmetic:

<table>
<thead>
<tr>
<th>Coins</th>
<th>PP</th>
<th>PN</th>
<th>DP</th>
<th>DN</th>
<th>DD</th>
<th>QP</th>
<th>QN</th>
<th>QD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x) )</td>
<td>( \frac{6}{28} )</td>
<td>( \frac{4}{28} )</td>
<td>( \frac{8}{28} )</td>
<td>( \frac{2}{28} )</td>
<td>( \frac{1}{28} )</td>
<td>( \frac{4}{28} )</td>
<td>( \frac{1}{28} )</td>
<td>( \frac{2}{28} )</td>
</tr>
</tbody>
</table>

In the simplified table, we take the products of the columns and sum them to get \( E(X) = .135, V(X) = \frac{201}{7000} \) and \( \sigma_x \approx .10242. \)

15. From past experience it is known that 3% of accounts in a large accounting population are in error. Joe is given a randomized list of accounts to audit. What is the probability that he audits 5 accurate accounts before he finds the 6th account in error?

This is a geometric distribution, with \( P(X = x) = (.97)^x(.03). \) In this case, \( x = 5, \) so \( P(x = 5) = (.97)^5(.03) \approx 0.02576. \) This is slightly different than the answer from the group who found it, but it appears the author of the question meant for \( x = 4. \) (I changed the question to be very precise about what it was asking, and apparently disagreed with the author’s interpretation.)
16. An oil company conducts a geological study that indicates that an exploratory oil well in a certain region should have a 20% chance of striking oil. What is the probability that the first strike comes on the third well drilled? The company wants three working rigs in the region. What is the probability that they can have to drill 8 times or fewer to get the three they need?

This is a negative binomial distribution, but the first part can be done easily without knowing this: \( P(\text{Success on 3}) = (.8)^3 (.2) = .128 = 12.8\% \)

For the second part, a chart is extremely helpful. Let \( X \) be the simple random variable indicating the number of failures prior to the third success. Then we calculate:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( p(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>( \binom{3}{1} (.8)^1 (.2)^4 )</td>
</tr>
<tr>
<td>2</td>
<td>( \binom{3}{2} (.8)^2 (.2)^4 )</td>
</tr>
<tr>
<td>3</td>
<td>( \binom{3}{3} (.8)^3 (.2)^4 )</td>
</tr>
<tr>
<td>4</td>
<td>( \binom{3}{4} (.8)^4 (.2)^4 )</td>
</tr>
<tr>
<td>5</td>
<td>( \binom{3}{5} (.8)^5 (.2)^3 )</td>
</tr>
</tbody>
</table>

We get the required probability by summing the cases listed in the chart.

17. What is the probability of getting a license plate that has a repeated letter or digit if you live in a state where the license plate scheme is four letters followed by two numerals?

The question is ambiguous. Is the repeat something like DEEK? Or is DATA a repeated letter? And what about DADA? That has “a” repeated letter, as well. If we take the widest possible definition of “repeated,” any letter reappearing anywhere else, and allow multiple repeats, the question is actually not too hard.

We can calculate the probability of getting no repeats at all, and subtract from 1. Note that there are 10 2-digit number patterns where the digits repeat: 00, 11, 22, 33, \ldots, 99.

\[
P(\text{no repeats}) = \frac{\binom{26}{4} 4!}{26^4 \times 10^2} = \frac{1725}{2197} \approx .78516
\]

The required probability is approximately \( .21484 = 1 - .78516 \), which disagrees with the answer turned in by the group who found it. I’m not surprised, as I’m pretty unclear about which patterns the question-writer intended for us to include and which to exclude.

18. On a multiple-choice test of 10 questions, each question has 5 possible answers. A student is certain of the answers to 4 questions but is totally baffled by 6 questions. If the student randomly guesses the answers to those 6 questions, what is the probability
that the student will get a score of 5 or more on the test? Express your answer correct to two decimal places.

We can calculate a probability table for this binomial distribution, and pick off the terms we need afterwards. Let \( X \) be the simple random variable that counts the number of guesses that are accurate out of the 6 attempts.

<table>
<thead>
<tr>
<th>Test Score</th>
<th>( x )</th>
<th>( p(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0</td>
<td>( \binom{6}{0}(.2)^0(.8)^6 )</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>( \binom{6}{1}(.2)^1(.8)^5 )</td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>( \binom{6}{2}(.2)^2(.8)^4 )</td>
</tr>
<tr>
<td>70</td>
<td>3</td>
<td>( \binom{6}{3}(.2)^3(.8)^3 )</td>
</tr>
<tr>
<td>80</td>
<td>4</td>
<td>( \binom{6}{4}(.2)^4(.8)^2 )</td>
</tr>
<tr>
<td>90</td>
<td>5</td>
<td>( \binom{6}{5}(.2)^5(.8)^1 )</td>
</tr>
<tr>
<td>100</td>
<td>6</td>
<td>( \binom{6}{6}(.2)^6(.8)^0 )</td>
</tr>
</tbody>
</table>

19. On a multiple-choice test of 10 questions, each question has 5 possible answers. A student is certain of the answers to 4 questions but is totally baffled by 6 questions. If the student randomly guesses the answers to those 6 questions, find a probability density function (pdf) for the number of correct responses earned. Calculate the mean and standard deviation for the pdf.

The pdf was already created, as the table in the solution to the question above is a pdf in tabular form.

20. There are 3 urns each containing red and black marbles (see table below). You draw one marble from Urn 1. If you draw a red marble from Urn 1, you make your second draw from Urn 2. If you draw a black marble from Urn 1, you make your second draw from Urn 3. What is the probability of drawing two marbles of the same color?

<table>
<thead>
<tr>
<th>Urn</th>
<th>Red Marbles</th>
<th>Black Marbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

We can get the same color by drawing BB or RR, so

\[
P(\text{same color}) = \frac{9}{10} \cdot \frac{5}{10} + \frac{1}{10} \cdot \frac{7}{10} = \frac{45}{100} + \frac{7}{100} = \frac{52}{100}
\]

21. Five number digits are generated in five rounds. In the first round, a digit from 0, ..., 9 could be selected, with equal probability. In each further round, if a1...an has already been selected, then the next number selected could only be n+1, ..., 9 with all these choices having equal probability. In other words, the digits of the number generated will always increase, like in 35689 or 24789. Find the probability that a sequence:
(a) terminates after 1, 2, 3 or 4 rounds (e.g. a 9 is selected and, therefore, the process ends).
(b) given a 5-digit is generated, that it consists of all even integers.
(c) consists of all odd integers.

22. In Franklin College, 40% of the freshmen are enrolled in a mathematics course, and 75% are enrolled in an English course, and 20% are taking both.

(a) What is the probability that a randomly selected freshman is taking an English course if it is known that he or she is enrolled in a mathematics course?
(b) If a randomly selected freshman is taking an English course, what is the probability that he or she is also enrolled in a mathematics course?

23. Eight tennis players (call them A,B,C,D,E,G,F,H) are randomly assigned to start positions in a ladder tournament. Initially, position 1 plays position 2, position 3 plays 4, 5 plays 6 and 7 plays 8. Second round has 2 matches: winner of (1,2) match plays winner of (3,4), and winner (5,6) plays winner(7,8). The winners of the two 2nd round matches play each other in the final match. Player A wins against any of the others. Player B always beats any opponent except player A. What is the probability that player B wins the 2nd place trophy in the final match?

24. In a roomful of 30 people, what is the probability that at least two people have the same birthday? Assume birthdays are uniformly distributed and there is no leap year complication.

Google this - it’s a classic problem, and several innovative presentations of the proper calculations are available. Be sure to verify the accuracy of the solution(s) you find!!

25. A coach is training 15 girls. He wants to form 5 lines of 3 forwards each (left-wing, center, and right-wing). Assume that the order of assigning these positions matters. What is the probability that both Ann and May are in the same line?

26. What is the probability of receiving a 7 (3 letters, then 4 numbers) digit license plate with a repeated letter/number?

27. What is the probability of having a license plate (3 letters, then 4 numbers) with either all vowels OR consonances?

28. What is the probability of randomly selecting a bill from a wallet and getting a $20 bill out of a wallet with 2 tens, 3 fives, 4 twenties, and 7 ones?

29. What is the probability of getting a 70% or better on a 20 question multiple choice test with 4 choices each, randomly guessing?
30. A certain school has three exam slots per day and 4 days for finals. Assuming exams are scheduled at random, what is the probability of a student with 5 classes having 3 finals scheduled on a single day?

31. What is the probability of drawing a red or blue marble out of a bowl with 10 red, 6 blue, 9 green?

32. What is the probability of rolling snake eyes?

33. What is the probability that a five-card poker hand will contain at least one of each suit?

34. When selecting three cards in an ordered sequence, what is the probability that the rank of the first card is strictly smaller than the rank of the second card which, in turn, is strictly smaller than the rank of the third card?

35. What is the probability of drawing an Ace then a ♥ and then the 3♣?

36. A die is rolled, find the probability that an even number is obtained.

37. Two dice are rolled, find the probability that the sum is
   (a) equal to 1
   (b) equal to 4
   (c) less than 13

The following example of an “ordered list” is helpful for the “Monopoly Dice Roll” where two standard 6-sided dice are rolled, and their values summed:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
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<td>7</td>
<td>8</td>
<td>9</td>
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<td>11</td>
</tr>
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<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

Let R be the value of the dice roll. Since each value is equally likely, we see that \( P(R = 4) = \frac{3}{36} = \frac{1}{12} \) (for part b above), and \( P(R = 1) = 0, P(R \leq 13) = 1 \) for the other questions.

38. A card is drawn at random from a deck of cards. Find the probability of getting the 3♦.

39. A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.
40. A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If two marbles are drawn from the jar at random, what is the probability that one marble is white and one is red?

41. A die is rolled. What is the probability that the number is even or less than 4?

42. You roll two 6-sided dice. What is the probability that the sum of the two dice is 10?

43. An identification code is to consist of seven letters followed by three digits. How many different codes are possible if repetition is permitted?

44. An 8-bit binary word is a sequence of 8 digits, of which each may be either a 0 or a 1. How many different 8-bit words are there?

45. Jane and Thomas are among the 8 people from which a committee of 4 people is to be selected. How many different possible committees of 4 people can be selected from these 8 people if at least one of either Jane or Thomas is to be selected?

46. Georgia license plates consist of three letters followed four numbers. How many license plate combinations are possible if the license plate contains no vowels (A, E, I, O, U, or Y) nor contains the number 5.

47. Given that there are four characters (Adenine, Guanine, Cytosine, Thymine) in 25 character strand of DNA, what is the chance that a poly-atail of twenty five adenines can occur in sequence if they occur randomly in nature?

48. Thee eleven tile letters PENULTIMATE are in a bag. Find the probability that, if 8 letters are drawn at random and laid down in order, the word “ultimate” is spelled?

49. A fair spinner has four equally divided color sections (blue, red, green, and yellow). What is the probability that the spinner will land on green?

50. An urn contains 5 red, 3 green, and 4 yellow marbles. A marble is randomly chosen from the urn. The marble is replaced in the urn, and a second marble is chosen. What is the probability of choosing a red and then a purple marble?

51. A class has 4 boys and 12 girls. They are randomly divided into 4 groups of 4. What is the probability that each group has 1 boy?

52. In Franklin College, 40% of the freshmen are enrolled in a mathematics course, and 75% are enrolled in an English course, and 20% are taking both.

   (a) What is the probability that a randomly selected freshman is taking an English course if it is known that he or she is enrolled in a mathematics course?

   (b) If a randomly selected freshman is taking an English course, what is the probability that he or she is also enrolled in a mathematics course?
(c) Are the two events to be enrolled in an English course, and to be enrolled in a mathematics course independent events for freshman?

53. You roll a standard 6-sided die and draw a letter from a bag containing the 26 tiles, each with a different letter of our standard English alphabet on it. What are your chances of picking:

(a) a 2 and an M?
(b) an even number and a consonant (not counting “Y”)?

54. A new health restaurant is being opened that randomly chooses 9 healthy items of food from 10 different vegetables and 15 different fruits to make a smoothie out of. What is the probability that at least 6 of the healthy food items are vegetables?

55. What is the probability of rolling 2 dice and drawing a card from a deck without aces or face cards and getting the same number?

56. A class of 20 boys and 10 girls has 15 Math majors and 15 Physics students. What is the probability of picking a student who is both a boy and a physics major? A physics major and a girl? A girl and a physics major?

57. Two students from Class A transferred into Class B. Before the transfer, Class A had 15 Math Students, 4 Physics Students and 1 Chemistry major Class B had 20 Physics students, and 5 Math students. What is the probability of finding a math major in class B after the transfer? Chemistry? Physics?

58. A teacher keeps a jar full of different flavored jelly beans on her desk and hands them out randomly to her class. But one particularly picky student likes only the licorice-flavored ones. If the jar has 50 beans in all 15 licorice, 10 cherry, 20 watermelon, and 5 blueberry. What is the probability that the first three jelly beans picked out are licorice flavored?

59. If a five-letter word is formed at random (meaning that all sequences of five letters are equally likely), what is the probability that no letter occurs more than once?

60. In a game of poker, what is the probability that a 5-card hand will contain a 4-of-a-kind?

Let 4K be the event of drawing a 4-of-a-kind to a 5-card poker hand drawn at random from a standard deck of 52 cards. Then

\[
P(4K) = \frac{\binom{13}{1} \binom{4}{1} \binom{48}{1}}{\binom{52}{5}}
\]

Let’s discuss the task that each of those terms is performing:

- \( \binom{13}{1} \) choosing a value for the 4-of-a-kind
• \( \binom{4}{4} \) choosing all 4 of those value
• \( \binom{48}{1} \) choosing any other card to be the 5th card in our hand
• \( \binom{52}{5} \) total number of poker hands possible

61. Suppose a pair of dice are thrown, and then thrown again. What is the probability that the sum appearing on the second throw is the same as the first?

Consider the problem of rolling a single die. What is the probability of seeing the second throw match the first? \( \frac{1}{6} \). Why? Whatever is rolled first, there is a \( \frac{1}{6} \) chance of rolling that value on the second throw.

The actual question we are asked requires a weighted average, since the first rolls occur in varying frequencies. Here’s a table with the probabilities for all 11 possibilities:

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x) )</td>
<td>( \frac{1}{36} )</td>
<td>( \frac{3}{36} )</td>
<td>( \frac{1}{36} )</td>
<td>( \frac{5}{36} )</td>
<td>( \frac{1}{36} )</td>
<td>( \frac{5}{36} )</td>
<td>( \frac{1}{36} )</td>
<td>( \frac{3}{36} )</td>
<td>( \frac{1}{36} )</td>
<td>( \frac{2}{36} )</td>
<td>( \frac{1}{36} )</td>
</tr>
</tbody>
</table>

So, the one-twelfth of the time we roll a 2, we have a one-twelfth chance of rolling it again. It should be clear if you think about it, the correct answer will be generated by squaring the probabilities in the second row of the table, and then summing them.

62. The letters of the word ABACUS are to be rearranged at random. What is the probability that the vowels all appear together?

63. Suppose we know there are three tails and two heads, what is the probability that the order was exactly HTHTT?

64. The letters of the word MEDITERRANEAN are to be rearranged at random. Find the probability that first letter is E and the last letter is R.

65. There are three urns with marbles in them. In the first urn there are 4 red marbles, 3 green marbles, and one pink ball. In the second urn there are 7 orange marbles, 3 blue marbles and 2 pink marbles. In the third urn there are 4 green marbles, 4 red marbles, 3 brown marbles, and 2 pink marbles. What is the probability of getting exactly two pink marbles if you pick two marbles out of each urn?

<table>
<thead>
<tr>
<th>Urn 1</th>
<th>4 Red</th>
<th>3 Green</th>
<th>1 Pink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urn 2</td>
<td>7 Orange</td>
<td>3 Blue</td>
<td>2 Pink</td>
</tr>
<tr>
<td>Urn 3</td>
<td>4 Red</td>
<td>3 Green</td>
<td>3 Brown</td>
</tr>
</tbody>
</table>

66. In a bag there is a tile with every letter of the alphabet in it, with two of each vowel (including Y), and 5 B tiles and 4 S tiles. If picking 11 tiles at random what is the probability of pulling out the tiles in the order to spell PROBABILITY?
<table>
<thead>
<tr>
<th>Letter</th>
<th>Number Left</th>
<th>Total Letters Left</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1</td>
<td>39</td>
<td>$\frac{1}{39}$</td>
</tr>
<tr>
<td>R</td>
<td>1</td>
<td>38</td>
<td>$\frac{1}{38}$</td>
</tr>
<tr>
<td>O</td>
<td>1</td>
<td>37</td>
<td>$\frac{1}{37}$</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>36</td>
<td>$\frac{1}{36}$</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>35</td>
<td>$\frac{1}{35}$</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>34</td>
<td>$\frac{1}{34}$</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>33</td>
<td>$\frac{1}{33}$</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>32</td>
<td>$\frac{1}{32}$</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>31</td>
<td>$\frac{1}{31}$</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
<td>30</td>
<td>$\frac{1}{30}$</td>
</tr>
<tr>
<td>Y</td>
<td>1</td>
<td>29</td>
<td>$\frac{1}{29}$</td>
</tr>
</tbody>
</table>

The solution is the product of the entries in the final column.

67. You’re playing five card stud poker. You’re starting hand is 2♥, 4♠, 6♦, J♥, and K♥. When it is time to discard, you are in a dilemma. Either discard the 4S and the 6♦ and try for a flush in ♥’s, or discard the J♥ and the K♥ in hopes of getting a 6-high straight. Which has the higher probability?

68. In a game of Yahtzee you are allowed to role 5 dice the first role, and however many you want for the second and third roles. You need a yahtzee (all 5 die read the same number). If you role every dice for all three roles what are the chances of getting yahtzee?

69. Two couples and one single person are seated at random in a row of five chairs. What is the probability that neither of the couples sits together in adjacent chairs?

70. What is the probability a license plate with 3 letters and 4 digits has the word dog in it?

71. What is the probability that a license plate 3 letters and 4 digits has an L as the last letter and a 5 as the first number?

72. Proteins are made up of chains of amino acids. Insulin is a relatively small protein with 53 amino acid residues. How many possible proteins of length 53 can be made with 20 possible amino acids for each position in the protein?

73. In a 20 sequence strand of DNA, what is the probability that the first 5 bonding sequences must be A-T, G-C, C-G, A-T, T-A? (Assuming that A always bonds with T and G always bonds with C).

74. Ishmael writes a computer program that produces at random one of these digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. What is the probability that the program produces:
(a) an even number
(b) a multiple of 4
(c) a number less than 7
(d) a multiple of 5

75. In Hannah’s purse there are three 1p coins, five 10p coins and eight 2p coins. She takes a coin at random from her purse. What is the probability of:

(a) a 1p coin
(b) a 2p coin
(c) not a 1p coin
(d) a 1p coin or 10p coin

76. Some of the children in a class write down the first letter of their surname on a card. These cards are shown below:

<table>
<thead>
<tr>
<th>W</th>
<th>S</th>
<th>M</th>
<th>E</th>
<th>G</th>
<th>S</th>
<th>A</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>H</td>
<td>T</td>
<td>S</td>
<td>E</td>
<td>I</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

One of these cards is taken at random.

(a) What is the probability that the letter on it is:
   i. W
   ii. S or T
   iii. J or M
   iv. not H
   v. a vowel

(b) Which letter is the most likely to be chosen?

77. A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

78. At the Tire Store a problem has occurred unbeknownst to the managers: 10% of all the tires currently in stock in their warehouse are defective. Mr. Z purchase 4 tires each for two vehicles and they are randomly selected from the stock. Find the probability that:

(a) all 8 tires will be defective?
(b) none of the tires will be defective?
(c) at least one of the tires will be defective?
(d) 3 or more of the tires will be defective?

79. What is the probability that you will have a license plate with the letters KC and the last number be 4, if there are two letters and four numbers on each license plate?

80. A spinner is divided into five equal sections numbered 1 through 5. The arrow is equally likely to land on any section. Find the probability of:

(a) an odd number on any one spin.
(b) at least three odd numbers on four spins.
(c) at least two odd numbers on four spins.
(d) at least one odd number on four spins.

81. Sara is rolling a die, what the probability that she rolls a 3 on the 4th roll?

82. Steph is a North Georgia basketball player and a 75% free throw shooter. During the season, find the probability that:

(a) She makes her third shot on her fifth attempt.
(b) She makes at least 8 of her first 10 attempts.
(c) She makes 7 shots before missing her second.

83. A multiple choice test contains 10 questions. Each question has five choices for the correct answer. Only one of the choices is correct.

(a) What is the probability of making exactly 80% with random guessing?
(b) What is the probability of passing, e.g. making at least 60% with random guessing?

84. Two girls and their dates go to the drive-in, and each wants a different flavored ice cream cone. The drive-in has 24 flavors of ice cream. How many combinations of flavors may be chosen among the four of them if each one selects one flavor?

85. We want to paint three rooms in a house, each a different color, and we may choose from seven different colors of paint. How many color combinations are possible for the three rooms?